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Chaotic vibrations of a nonideal electro-mechanical system

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Abstract

Nonideal systems are those in which one takes account of the influence of the oscillatory system on the energy supply with a limited power (Kononenko, 1969). In this paper, a particular nonideal system is investigated, consisting of a pendulum whose support point is vibrated along a horizontal guide by a two bar linkage driven by a DC motor, considered to be a limited power supply. Under these conditions, the oscillations of the pendulum are analyzed through the variation of a control parameter. The voltage supply of the motor is considered to be a reliable control parameter. Each simulation starts from zero speed and reaches a steady-state condition when the motor oscillates around a medium speed. Near the fundamental resonance region, the system presents some interesting nonlinear phenomena, including multi-periodic, quasiperiodic, and chaotic motion. The loss of stability of the system occurs through a saddle-node bifurcation, where there is a collision of a stable orbit with an unstable one, which is approximately located close to the value of the pendulum's angular displacement given by $\alpha_C = \pi/2$. The aims of this study are to better understand nonideal systems using numerical simulation, to identify the bifurcations that occur in the system, and to report the existence of a chaotic attractor near the fundamental resonance. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Nonideal systems; Nonlinear dynamics; Chaotic vibrations

1. Introduction

In the last two decades, in the field of nonlinear dynamics, there has been an increasing number of research works through analysis of different mathematical models, and physical experiments. These research works describe the behavior of nonlinear dynamic systems and identify and analyze its bifurcations. Among the steady-state solutions that may appear in such systems, chaos has received the greatest attention, mainly in the identification of the interesting forms of its strange attractors, a topic dealt with in mathematical topology. Mechanisms of routes to chaos also are of great interest since they define the way the system loses stability near a bifurcation point. Among these mechanisms are intermittency, period

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doubling cascade, and quasiperiodicity. Actually, the interest in these phenomena lies in discovering mechanisms to control the system near the nonlinear resonance region, with the aim of obtaining a desired behavior inside an unstable domain of the chosen parameters.

However, in majority of these studies, the mathematical modeling is done following the assumption that the external driving force does not experience any influence from the oscillatory system, and the energy source works without any internal interference. These systems are called systems with an ideal energy source, where it is assumed that the force's amplitude and its frequency are arbitrary constants. But this simplification may not completely explain all the system properties since due to the mechanical connection between the energy source and the oscillating system, a dynamic interaction will happen.

A more realistic formulation is to consider an energy source with limited power (nonideal), i.e., to consider the influence of the oscillatory system on the driving force and vice versa (Nayfeh and Mook, 1979). A distinctive property of a nonideal energy source is that it cannot be described by a certain function that varies in time because its action also depends on the motion of the oscillatory system. Its motion, governed by a differential equation, must be included in the dynamic system, and this will increase the number of degrees of freedom. In the formulation of the rotating shaft mechanism to study the passage through one of its critical speeds (Suherman and Plaut, 1997), the nonideal supposition can be used. Also, it can be used in mechanisms where transient motions have a crucial importance in the determination of final results since this supposition considers the dynamic interactions in the system, and consequently, it increases the quality of the numerical solutions.

In this study, we analyze a particular nonideal, nonlinear dynamic system consisting of a simple pendulum whose support point is vibrated along a horizontal guide by a two bar linkage driven by a DC motor with limited power (Fig. 1). With this supposition, the pendulum's support point is vibrated by a periodic nonharmonic force because it undergoes the action from the motor and the influence of the pendulum's oscillation. The simple pendulum is a classical problem in nonlinear dynamics providing a great number of nonlinear phenomena; we will describe its behavior considering the complex interaction with a limited energy source.

Kranospol'kaya and Shvets (1990,1993) first investigated this particular system through the analysis of three nonlinear coupled, averaged equations and referring to this system as "electromotor–pendulum" system. They identified, in their work, two chaotic regions close to fundamental resonance, and the routes to chaos were intermittency and period doubling cascade.

This paper is organized as follows: Section 2 contains a description of the mathematical model of the electromotor–pendulum, and Section 3 presents the main numerical results of this system near the fundamental resonance region and an analysis of the results.

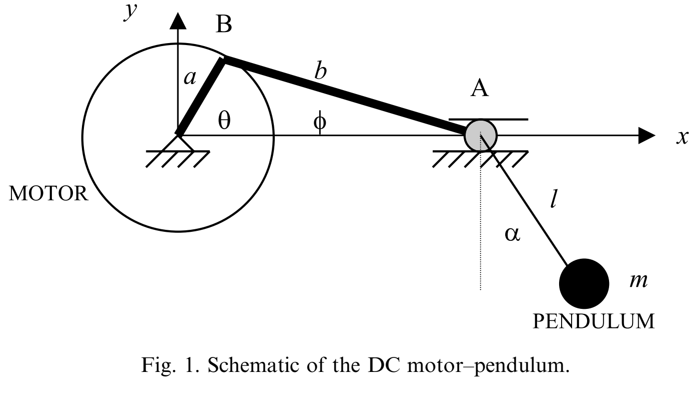

Fig. 1. Schematic of the DC motor–pendulum.

2. Equations of motion

The mechanism consists of a pendulum whose support point A is displaced along a horizontal guide by a DC motor. When the load motor rotates at an angle θ, the horizontal displacement of pin A is $s_A(t) = a\cos\theta + b(1-\epsilon_1^2\sin^2\theta)^{1/2}$, where a is the length of the crank rod, b is the length of the crank mechanism and the ratio $\epsilon_1 = a/b$ is generally a small parameter.

The kinetic and potential energies are defined as

$$\mathsf{K} = \tfrac{1}{2}J\dot{\theta}^2 + \tfrac{1}{2}m[(-a\dot{\theta}F + l\dot{\alpha}\cos\alpha)^2 + (-l\dot{\alpha}\sin\alpha)^2], \tag{1}$$

$$\mathsf{V} = mgl(-\cos\alpha) \tag{2}$$

with

$$F = \left[1 + \frac{\epsilon_1\cos\theta}{\left(1-\epsilon_1^2\sin^2\theta\right)^{1/2}}\right]\sin\theta, \tag{3}$$

where $\dot{\theta}$ is the motor's speed, α is the pendulum's angular displacement, J is the mass moment of inertia of the rotor, m is the pendulum's mass and l is the length of the pendulum. We use the Lagrangian function, defined as $\mathsf{L} = \mathsf{K} - \mathsf{V}$, to obtain the equations of motion for the electromotor–pendulum system.

The motion of the DC motor is governed by the following equations:

$$\begin{aligned} L\dot{I}(t) &= V(t) - RI(t) - K_E\dot{\theta}(t), \\ M_{MOTOR} &= K_T I(t) - c_m\dot{\theta} - T_f, \end{aligned} \tag{4}$$

where $V(t)$ is the motor voltage, I is the current, R is the electric resistance, L is the armature inductance, K_T is the torque constant, K_E is the voltage constant, c_m is the constant for the internal loss coefficient in the motor and T_f is the constant friction torque in the motor. The first equation is called the *electrical equation* of the motor, and the second one defines the torque generated by the motor, M_{MOTOR}.

We will use a simplified model of a DC motor, where the permanent magnetic field can be derived by assuming zero armature inductance. Considering also that $T_f = 0$ and that the *electrical time constant of the motor* L/R is smaller than the *mechanical time constant* $RJ/K_E K_T$, the simplified DC motor's equations becomes

$$\begin{aligned} V(t) &= RI(t) + K_E\dot{\theta}(t), \\ M_{MOTOR} &= K_T I(t) - c_m\dot{\theta}. \end{aligned} \tag{5}$$

Then, using the Lagrangian function L, we obtain the system equations,

$$\left(J + ma^2F^2\right)\ddot{\theta} - maF(l\cos\alpha\ddot{\alpha} - a\dot{F}\dot{\theta} - l\sin\alpha\dot{\alpha}^2) = G_\theta, \tag{6}$$

$$m(l^2\ddot{\alpha} - alF\cos\alpha\ddot{\theta} - al\dot{F}\cos\alpha\dot{\theta} + gl\sin\alpha) = G_\alpha, \tag{7}$$

where $\dot{F} = \mathrm{d}F/\mathrm{d}t$, and G_θ, G_α are generalized forces given by

$$G_\theta = M_{MOTOR} - \mu_A a^2F^2\dot{\theta} + \frac{a}{l}\mu_l F\dot{\alpha}\cos\alpha, \qquad G_\alpha = -\mu_l\dot{\alpha}.$$

The constants μ_A, μ_l define, respectively, the viscous damping coefficient at the rolling pin A (in the x direction) and the damping coefficient of the pendulum.

Using the transformation, $t^* = \omega_0 t$, where t^* is a dimensionless time and ω_0 is the natural frequency of the pendulum, the equations of motion of the mechanism can be written as (Belato, 1998)

$$\left(J+\beta_4F^2\sin^2\alpha\right)\theta''=\beta_1-\left(\beta_2+\beta_3F^2\right)\theta'-\beta_4\sin^2\alpha FF'\theta'-\beta_5F\left(\cos\alpha+\alpha'^2\right)\sin\alpha, \tag{8}$$

$$\alpha''+\sin\alpha=\epsilon_2\left(F\theta''+F'\theta'\right)\cos\alpha-\beta_6\alpha', \tag{9}$$

where

$$F=\left[1+\frac{\epsilon_1\cos\theta}{\left(1-\epsilon_1^2\sin^2\theta\right)^{1/2}}\right]\sin\theta,\qquad F'=\frac{\mathrm{d}F}{\mathrm{d}t^*},\qquad \beta_1=\frac{K_\mathrm{T}V(t)}{R\omega_0^2}$$

is the control parameter,

$$\beta_2=\frac{K_\mathrm{E}K_\mathrm{T}}{R\omega_0}+\frac{c_\mathrm{m}}{\omega_0},\qquad \beta_3=\frac{\mu_\mathrm{A}a^2}{\omega_0},\qquad \beta_4=ma^2,\qquad \beta_5=mal,\qquad \beta_6=\frac{\mu_l}{\omega_0ml^2},\qquad \epsilon_2=\frac{a}{l},$$

and the primes denote derivatives with respect to t^*.

3. Numerical results

Numerical simulations are done in Matlab™, adopting β_1 as a control parameter and $\beta_2=0.02448$ kgm^2, $\beta_3=0$, $\beta_4=0.00098$ kgm^2, $\beta_5=0.0042$ kgm^2, $\beta_6=0.01$, $\epsilon_1=\epsilon_2=0.2333$, $J=0.001655$ kgm^2 with the initial conditions $\alpha(0)=\alpha'(0)=0$ and $\theta(0)=\theta'(0)=0$. The numerical integrator is the Runge Kutta fifth-order algorithm with variable time step.

It is known that the dynamics of a system close to the fundamental resonance region may be analyzed through a frequency–response diagram, which is obtained plotting the amplitude of the oscillating system versus the frequency of the excitation term. For the electromotor–pendulum, this graph is estimated by numerical simulation defining the amplitude as the maximum absolute value of the amplitude of the pendulum's oscillation (denoted by $|\alpha_\mathrm{M}|$), and the frequency as the mean value of the rotational speed of the motor θ' (denoted by ω).

Fig. 2 represents the resonance curve when the mean frequency ω is slowly increased. The results are similar when the mean frequency ω is slowly decreased. The curve was calculated using an increment $\Delta\beta_1=0.00001$ as the variation of the control parameter β_1. The transient response is also considered in the computation because its evolution inside the state space determines the occurrence of the jump phenomenon. This dependence on the system's transient behavior was also observed by McRobie and Thompson (1992) for the capsize of a craft.

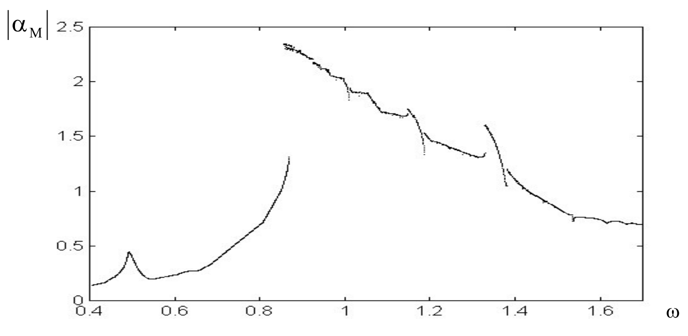

Fig. 2. Frequency–response diagram: Jump phenomenon observed when the mean frequency ω is slowly decreased or increased.

For the chosen parameter, no change in the curve's shape, mainly near the jump region when the mean frequency ω is increased or decreased, was observed. In this region, the process occurs the same way in both cases: there is a transition from a nonresonant to a resonant response (or vice versa) accompanied by a subsequent decrease in the mean frequency ω. This behavior occurs because the chaotic state of the pendulum puts a greater effort on the motor's operation, consequently, causing a small decrease in its mean frequency.

The dynamic analysis of this system may be compared with engineering systems that exhibit softening characteristics, where there is a discontinuous jump from a nonresonant to a resonant solution. However, in the present case, the resonant solution exhibits a more complicated and different behavior, confirmed by the presence of "secondary jumps" in the graph. These jumps cause an increase in the amplitude of pendulum's oscillation, followed by a steady-state synchronized motion (multi-periodic solution). To draw a comparison between this fact and the ideal systems studied previously, the frequency–resonance diagram only presents a decreasing smooth curve for the resonant solution; this indicates a diminution of the size of its basin of attraction. Some works also relate the existence of the synchronized motions in these systems, but no change in the aspect of their resonant curve was detected.

In a closer analysis of this curve, periodic solutions of the pendulum are observed when the value of the control parameter is approximately $\beta_1 \approx 0.02155$ kg m^2 ($\omega \approx 0.87$). Under a small increase of the control parameter's value, the system loses stability at a saddle-node bifurcation, which is associated with the "jump" phenomenon. This phenomenon occurs due to the presence of an unstable orbit (*saddle*) inside the potential well, which is approximately determined by the value of the pendulum's angular displacement $\alpha_C = \pi/2$. This saddle appears in the system due to the horizontal excitation of the pendulum, and it separates the interior of the pendulum's phase portrait into two different regions. This phase portrait is delimited by the heteroclinic orbits calculated by the unforced and undamped pendulum's equation. However, for the nonideal case, not only do the softening characteristic reveals the existence of two steady-state periodic solutions separated by this unstable orbit, but there is also a variation in the pendulum's oscillation in the solutions because when the pendulum loses stability, it is unable to settle on one of these two states. This phenomenon appears in the electromotor–pendulum system due to the adopted values $l = 0.3$ m, $a = 0.07$ m, $b = 0.3$ m, and if these values are diminished ($l = 0.07$ m, $a = 0.03$ m, $b = 0.2$ m), the pendulum manages to escape from the potential well.

Because of the values of the parameter chosen for the pendulum and the nonideal supposition, the system is not able to escape from the potential well when it loses stability at a saddle-node bifurcation. The stable attractor collides with the unstable orbit, and the system undergoes a jump to a greater attractor, located between the unstable orbit and the heteroclinic orbit. However, it does not settle on the greater periodic attractor, returning instead to the interior of the state space with minimum amplitude. This change in the pendulum's behavior reveals the existence of an intermittent limited process. [1] When these intermittent stages of greater (bursts) and smaller amplitude become more and more frequent, a chaotic attractor appears in the state space, which is confined inside a single potential well. In Fig. 3a, the chaotic attractor is given in the three-dimensional Poincare map, which is obtained every time the system's trajectory crosses $\theta = 0$, and Fig. 3b represents the projection of the map on the plane $\alpha' \times \alpha$.

When the value of the control parameter is increased beyond the chaotic solutions' domain, there is a variation among synchronized and nonsynchronized states culminating in a quasiperiodic motion when $\beta_1 = 0.03715$ kg m^2 ($\omega \approx 1.515$). In this domain of the control parameter, there occurs a small waving in the amplitude of the resonant response, increasing due to the proximity of the unstable orbit, located near the values $\alpha_C = \pi/2$. When the mean frequency is increased and the amplitude of the resonant curve diminishes

[1] Here, intermittent limited process means that the solution does not escape from the potential well determined by the minimum point $(\alpha, \alpha') = (0, 0)$, i.e., it is confined inside this potential well.

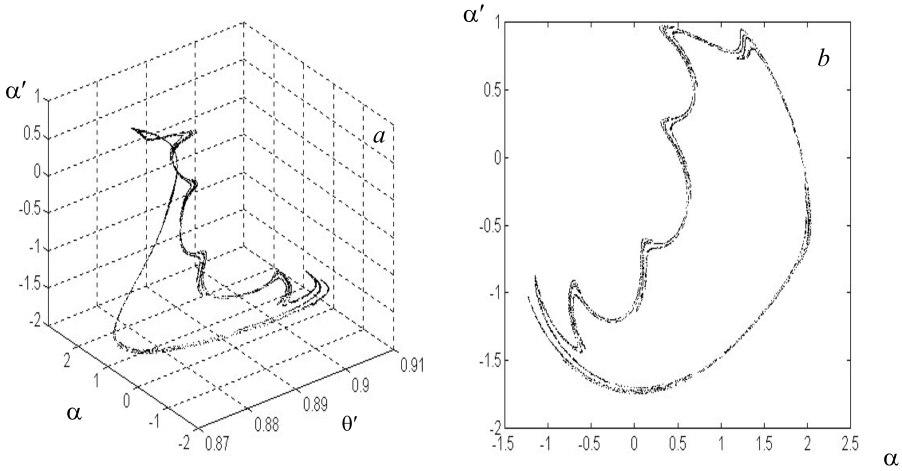

Fig. 3. The single well chaotic attractor obtained for $\beta_1 = 0.0217$ kgm^2: (a) three-dimensional Poincare map and (b) projection of the map on the plane $\alpha' \times \alpha$.

reaching the unstable orbit, a small jump occurs in the pendulum's solution when $\beta_1 = 0.02842$ kgm^2 ($\omega \approx 1.149$) and again when $\beta_1 = 0.03278$ kgm^2 ($\omega \approx 1.331$). In the first case, the jump is followed by a synchronized regime given by three periodic solutions, as observed in Fig. 4b; in the second case, a two periodic solution appears, as observed in Fig. 4d. This diversity of the amplitude is measured by the proximity of the unstable cycle, where the system searches for a more stable condition of oscillation given by the synchronization of the response.

In Fig. 4, we observe that the amplitude and the solution at the penultimate jump on the resonant curve, when $\beta_1 = 0.02842$, is smaller than the last one when $\beta_1 = 0.03278$. This fact occurs because the first one is closer to the unstable cycle. In this region of the resonance curve, these jumps cause a better arrangement of the frequency of both systems (the motor and the pendulum), and the system has a tendency to work with lesser effort. The synchronized solutions concentrate on the approximately vertical parts of the resonant curve, and the nonsynchronized solutions concentrate on the horizontal ones. In this domain of control parameter, a diminution of its basin of attraction with the increase of the mean frequency ω can be observed.

In this work, we only identified the domains of the control parameter near the fundamental resonance region where a different system's behavior must occur when the mean frequency of the motor is increased. Future studies should be carried out to provide a more complete analysis of the phenomena that appear along the frequency–response curve and to understand the way the system loses stability when the mean frequency is decreased.

4. Conclusions

A particular nonideal dynamic system has been analyzed through numerical simulation. An estimation is given of the critical value of the control parameter for which steady-state chaotic motion can be expected in the fundamental resonance region. Under nonideal conditions, the chaotic behavior of this system is characterized by a chaotic transient response that persists for a long time. Also, for some values of the control parameter, there are no detected differences among the transient and steady-state response. When the control parameter is increased, there is also a variation among synchronized states. The pendulum's

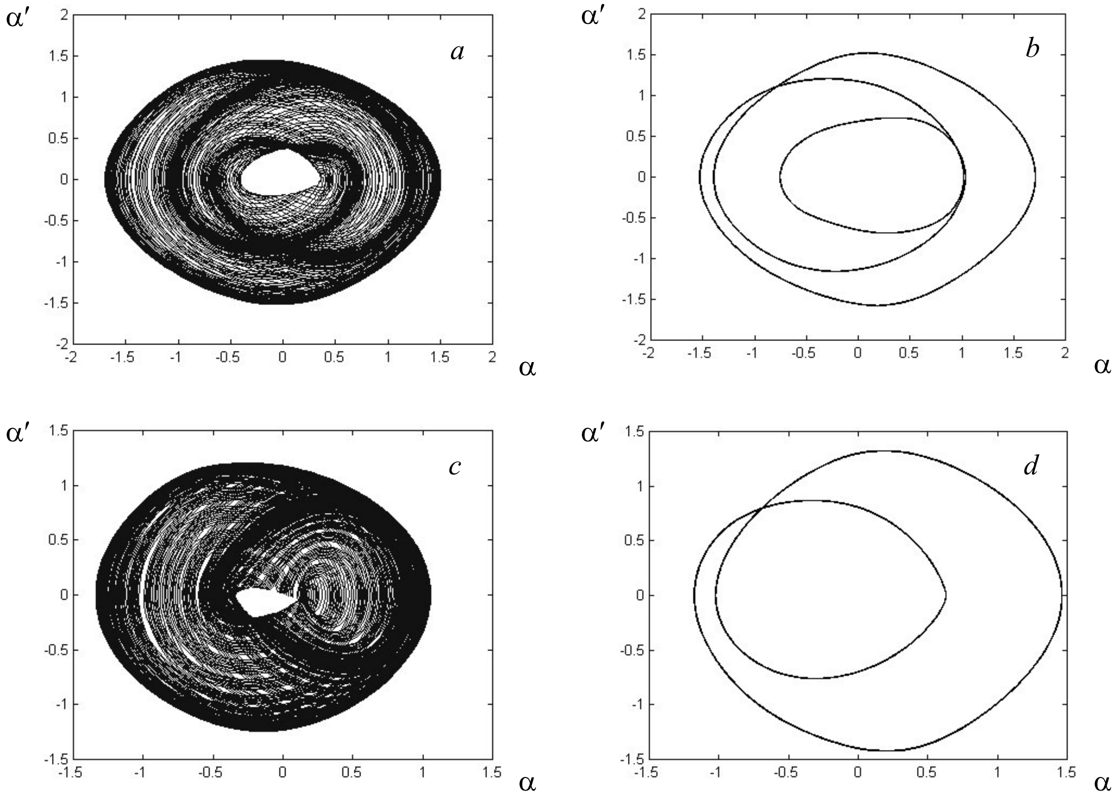

Fig. 4. State space of the pendulum when (a) $\beta_1 = 0.02841$ kgm^2 ($\omega \approx 1.15$), nonsynchronized state characterized by quasiperiodic solution; (b) $\beta_1 = 0.02842$ kgm^2 ($\omega \approx 1.149$), three-periodic solution; (c) $\beta_1 = 0.03277$ kgm^2 ($\omega \approx 1.332$), nonsynchronized state characterized by quasiperiodic solution and (d) $\beta_1 = 0.03278$ kgm^2 ($\omega \approx 1.331$), two-periodic solution. These solutions are obtained near the two last jump on the resonant curve.

solutions on the resonant curve undergo the interference of an unstable orbit, and when this becomes greater, there occurs a small jump that causes an increase in the amplitude pendulum's solution. This is followed by a steady-state, multi-periodic motion. When the mean frequency of the motor is increased, the transition to chaos is given by an intermittent phenomenon and the system loses stability through a saddle-node bifurcation.

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